

Remarks on Diffractive Dissociation within JIMWLK Evolution at NLO.

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We discuss the high energy diffractive dissociation in DIS at the Next to Leading Order. In the large N_c dipole limit we derive the NLO version of the Kovchegov-Levin equation. We argue that the original structure of the equation is preserved, that is it coincides with the Balitsky-Kovchegov equation at NLO.

I. INTRODUCTION AND CONCLUSION

In recent years a lot of attention has been devoted to development and phenomenological applications of the theory of perturbative saturation [1]. The theoretical description of the energy evolution of the wave function towards a dense state at leading order in α_s has been long known. It is given by JIMWLK equation [2, 3], or equivalently Balitsky hierarchy [4]. It generalizes the well known BFKL equation [5] by including finite density effects in the hadronic wave function. JIMWLK is restricted in applicability to DIS-type processes when target is large while projectile is small throughout the entire range in rapidity Y of the process. The mean field approximation to the JIMWLK equation, the Balitsky-Kovchegov (BK) equation [4, 6] has been used extensively during the last decade in numerous phenomenological applications, including fits to DIS low x data and the diffraction measured by HERA. The latter process is frequently regarded as a direct measure of non-linear high density effects. For JIMWLK-based phenomenological applications it is important to include perturbative corrections beyond leading order, since they are known to lead to large effects already in the linear BFKL framework [7].

Based on the milestone results of [8] and [9], in a recent paper [10] we have derived the complete operator form of the JIMWLK Hamiltonian at the next to leading order. This paper appeared simultaneously with [11], which directly calculated the elements of the general Balitsky hierarchy at NLO. In [12] we show that the NLO JIMWLK equation for $\mathcal{N} = 4$ theory has exact conformal invariance, even though it is derived with sharp rapidity cutoff.

At leading order, diffraction in saturation environment was explored by Kovchegov and Levin [13]. The diffractive cross section N_{El}^D investigated in [13] is diffractive on the projectile side but elastic in the target degrees freedom. It was shown in [13] that in the dipole limit, N_{El}^D obeys the BK equation. Running coupling effects [14] for the process have been included in [15].

The aim of the present note is to address diffractive dissociation within the NLO JIMWLK. In [16] we developed the formalism for calculating diffractive dissociation and other semi-inclusive observables within the JIMWLK framework. We considered several examples, in particular variety of elastic and/or diffractive cross sections, cross section with fixed transverse momentum transfer and inclusive gluon spectrum. In all these cases we defined the appropriate observable, derived its evolution with rapidity (total rapidity of the process and/or width of the rapidity gap and/or width of the diffractive interval) and discussed the dipole model limit for each one of the observables. Our main observation here is that many results of [16] are in fact independent of the explicit form of the JIMWLK Hamiltonian and thus generalize straightforwardly to the NLO accuracy.

The following two points summarize our findings. First, the expression for diffractive dissociation (3.1) which was originally derived in [16] is valid as long as the high energy factorization between projectile and target holds. Particularly it is valid, when the projectile is dilute, target is dense, and the respective evolution is governed by the NLO JIMWLK. The second point is that, just like at LO, the large N_c dipole Hamiltonian at NLO is linear in the dipole conjugate field. This makes the NLO version of the Kovchegov-Levin to coincide with the BK equation at NLO. In order to compute diffractive DIS cross sections, one needs to convolute N_{El}^D with the photon impact factor [17].

II. HIGH ENERGY SCATTERING

In this section we recap some basic formalism and introduce notations. The total S -matrix of the high energy scattering process at a given rapidity Y in the CGC formalism is computable via the following factorization formula

$$S(Y) = \int DS W_{Y_0}^T[S] \Sigma_{Y-Y_0}^{PP}[S]. \quad (2.1)$$

Here for a composite projectile which has some distribution of gluons in its wave function the eikonal S -matrix is

$$\Sigma_{Y-Y_0}^{PP}[S] \equiv \langle P | \hat{S} | P \rangle = \int d\rho W_{Y-Y_0}^P[\rho] \exp \left\{ i \int d^2x \rho^a(x) \alpha^a(x) \right\} \quad (2.2)$$

$\rho(x)$ is the color charge density in the projectile wave function at a given transverse position; $W_{Y-Y_0}^P[\rho]$ is its probability distribution in the projectile, while α is a target color field. In eq.(2.1), the projectile-averaged S -matrix Σ^{PP} is further averaged over the distribution of the color fields α with the weight $W_{Y_0}^T[S(x)]$. The fields α are parametrized by the eikonal S -matrix $S(x)$ for a single parton at transverse position x to scatter on a given configuration of α .

In Eq. (2.1) we have chosen the frame where the target has rapidity Y_0 while the projectile carries the rest of the total rapidity $Y - Y_0$. Lorentz invariance requires S to be independent of Y_0 . The high energy evolution of both $W^{P,T}$ is driven by an effective high energy Hamiltonian, which in this note will be assumed to be the NLO JIMWLK

$$\frac{d}{dY} W_Y^{P,T} = - H^{NLO \text{ JIMWLK}} W_Y^{P,T} \quad (2.3)$$

The NLO JIMWLK Hamiltonian [10] is

$$\begin{aligned} H^{NLO \text{ JIMWLK}} = & \int_{x,y,z} K_{JSJ}(x,y,z) [J_L^a(x) J_L^a(y) + J_R^a(x) J_R^a(y) - 2J_L^a(x) S_A^{ab}(z) J_R^b(y)] + \\ & + \int_{x,y,z,z'} K_{JSSJ}(x,y,z,z') [f^{abc} f^{def} J_L^a(x) S_A^{be}(z) S_A^{cf}(z') J_R^d(y) - N_c J_L^a(x) S_A^{ab}(z) J_R^b(y)] + \\ & + \int_{x,y,z,z'} K_{q\bar{q}}(x,y,z,z') [2J_L^a(x) \text{tr}[S^\dagger(z) T^a S(z') T^b] J_R^b(y) - J_L^a(x) S_A^{ab}(z) J_R^b(y)] + \\ & + \int_{w,x,y,z,z'} K_{JJSSJ}(w;x,y,z,z') f^{acb} [J_L^d(x) J_L^e(y) S_A^{dc}(z) S_A^{eb}(z') J_R^a(w) - \\ & - J_L^a(w) S_A^{cd}(z) S_A^{be}(z') J_R^d(x) J_R^e(y) + \frac{1}{3} [J_L^c(x) J_L^b(y) J_L^a(w) - J_R^c(x) J_R^b(y) J_R^a(w)]] + \\ & + \int_{w,x,y,z} K_{JJSSJ}(w;x,y,z) f^{bde} [J_L^d(x) J_L^e(y) S_A^{ba}(z) J_R^a(w) - J_L^a(w) S_A^{ab}(z) J_R^d(x) J_R^e(y) + \\ & + \frac{1}{3} [J_L^d(x) J_L^e(y) J_L^b(w) - J_R^d(x) J_R^e(y) J_R^b(w)]] \end{aligned} \quad (2.4)$$

Here S_A is a unitary matrix in the adjoint representation - the gluon scattering amplitude. The left and right $SU(N_c)$ rotation generators, when acting on functions of S have the representation

$$J_L^a(x) = \text{tr} \left[\frac{\delta}{\delta S_x^T} T^a S_x \right] - \text{tr} \left[\frac{\delta}{\delta S_x^*} S_x^\dagger T^a \right]; \quad J_R^a(x) = \text{tr} \left[\frac{\delta}{\delta S_x^T} S_x T^a \right] - \text{tr} \left[\frac{\delta}{\delta S_x^*} T^a S_x^\dagger \right]. \quad (2.5)$$

Here T^a are $SU(N_c)$ generators in the fundamental representation. We use the notations of ref. [8] $X \equiv x - z$, $X' \equiv x - z'$, $Y \equiv y - z$, $Y' \equiv y - z'$, $W \equiv w - z$, $W' \equiv w - z'$, and $Z \equiv z - z'$. All J s in (2.4) are assumed not to act on S in the Hamiltonian.

$$K_{JSJ}(x,y,z) = -\frac{\alpha_s^2}{16\pi^3} \frac{(x-y)^2}{X^2 Y^2} \left[b \ln(x-y)^2 \mu^2 - b \frac{X^2 - Y^2}{(x-y)^2} \ln \frac{X^2}{Y^2} + \left(\frac{67}{9} - \frac{\pi^2}{3} \right) N_c - \frac{10}{9} n_f \right] - \frac{N_c}{2} \int_{z'} \tilde{K}(x,y,z,z') \quad (2.6)$$

Here μ is the normalization point in the \overline{MS} scheme and $b = \frac{11}{3} N_c - \frac{2}{3} n_f$.

$$\begin{aligned} K_{JSSJ}(x,y,z,z') = & \frac{\alpha_s^2}{16\pi^4} \left[-\frac{4}{Z^4} + \left\{ 2 \frac{X^2 Y'^2 + X'^2 Y^2 - 4(x-y)^2 Z^2}{Z^4 [X^2 Y'^2 - X'^2 Y^2]} \right. \right. \\ & + \frac{(x-y)^4}{X^2 Y'^2 - X'^2 Y^2} \left[\frac{1}{X^2 Y'^2} + \frac{1}{Y^2 X'^2} \right] + \frac{(x-y)^2}{Z^2} \left[\frac{1}{X^2 Y'^2} - \frac{1}{X'^2 Y^2} \right] \left. \right\} \ln \frac{X^2 Y'^2}{X'^2 Y^2} \Big] + \tilde{K}(x,y,z,z') \end{aligned} \quad (2.7)$$

$$\tilde{K}(x,y,z,z') = \frac{i}{2} [K_{JJSSJ}(x;x,y,z,z') - K_{JJSSJ}(y;x,y,z,z') - K_{JJSSJ}(x;y,x,z,z') + K_{JJSSJ}(y;y,x,z,z')](2.8)$$

$$K_{q\bar{q}}(x, y; z, z') = -\frac{\alpha_s^2 n_f}{8\pi^4} \left\{ \frac{X'^2 Y^2 + Y'^2 X^2 - (x-y)^2 Z^2}{Z^4 (X^2 Y'^2 - X'^2 Y^2)} \ln \frac{X^2 Y'^2}{X'^2 Y^2} - \frac{2}{Z^4} \right\} \quad (2.9)$$

$$K_{JJJJ}(w; x, y; z) = -i \frac{\alpha_s^2}{4\pi^3} \left[\frac{X \cdot W}{X^2 W^2} - \frac{Y \cdot W}{Y^2 W^2} \right] \ln \frac{Y^2}{(x-y)^2} \ln \frac{X^2}{(x-y)^2} \quad (2.10)$$

$$K_{JJSSJ}(w; x, y; z, z') = -i \frac{\alpha_s^2}{2\pi^4} \left(\frac{X_i Y'_j}{X^2 Y'^2} \right) \left(\frac{\delta_{ij}}{2Z^2} - \frac{Z_i W'_j}{Z^2 W'^2} + \frac{Z_j W_i}{Z^2 W^2} - \frac{W_i W'_j}{W^2 W'^2} \right) \ln \frac{W^2}{W'^2} \quad (2.11)$$

Our goal will be to study diffractive dissociation at NLO within the dipole model approximation. The S -matrix for a quark-antiquark dipole is

$$s(x, y) = \frac{1}{N_c} \text{tr}[S_F(x) S_F^\dagger(y)] \quad (2.12)$$

where F denotes fundamental representation. The S matrix of a projectile made of several but not too many dipoles is therefore some function of the variable s only

$$\Sigma^{PP}[S] = \Sigma^{PP}[s]. \quad (2.13)$$

In the large N_c limit, the NLO JIMWLK Hamiltonian acting on a function of dipoles only, reduces to the action of the NLO dipole Hamiltonian. To derive this Hamiltonian, we have to act with the NLO JIMWLK Hamiltonian on one, two and three dipoles. Four and more dipoles cannot be coupled by the NLO evolution because there are only three J s in the Hamiltonian. The action on a single dipole is by construction [10] reproduces the result of [8] and, in the large N_c limit is equivalent to the action of the dipole Hamiltonian

$$\begin{aligned} H^{NLO \text{ dipole}} &= \frac{\alpha_s N_c}{2\pi^2} \int_{u,v} \left[\int d^2 z \frac{(u-v)^2}{U^2 V^2} \left\{ 1 + \frac{\alpha_s}{4\pi} \left[b \ln(u-v)^2 \mu^2 - b \frac{U^2 - V^2}{(u-v)^2} \ln \frac{U^2}{V^2} + \left(\frac{67}{9} - \frac{\pi^3}{3} \right) N_c - \right. \right. \right. \\ &\quad \left. \left. - \frac{10}{9} n_f - 2N_c \ln \frac{U^2}{(u-v)^2} \ln \frac{V^2}{(u-v)^2} \right\} [s(u, z)s(z, v) - s(u, v)] + \right. \\ &\quad \left. + \frac{\alpha_s^2}{16N_c \pi^4} \int d^2 z d^2 z' \left[\left(-\frac{4}{(z-z')^4} + \left\{ 2 \frac{U^2(V')^2 + (U')^2 V^2 - 4(u-v)^2(z-z')^2}{(z-z')^2 [U^2(V')^2 - (U')^2 V^2]} + \right. \right. \right. \\ &\quad \left. \left. + \frac{(u-v)^4}{U^2(V')^2 - (U')^2 V^2} \left[\frac{1}{U^2(V')^2} + \frac{1}{V^2(U')^2} \right] + \frac{(u-v)^2}{(z-z')^2} \left[\frac{1}{U^2(V')^2} - \frac{1}{(U')^2 V^2} \right] \right\} \ln \frac{U^2(V')^2}{(U')^2 V^2} \right) \times \\ &\quad \times [N_c^3 s(u, z')s(z', z)s(z, v) - (z' \rightarrow z)] \\ &\quad \left. - \left\{ \frac{(u-v)^2}{(z-z')^2} \left[\frac{1}{U^2(V')^2} + \frac{1}{V^2(U')^2} \right] - \frac{(u-v)^4}{V^2(U')^2(V')^2 U^2} \right\} \ln \left(\frac{U^2(V')^2}{(U')^2 V^2} \right) N_c^3 s(u, z')s(z', z)s(z, v) + \right. \\ &\quad \left. + N_c n_f \left\{ \frac{4}{(z-z')^4} - 2 \frac{U^2(V')^2 + (U')^2 V^2 - (u-v)^2(z-z')^2}{(z-z')^4 ((V')^2 U^2 - V^2 (U')^2)} \ln \left(\frac{U^2(V')^2}{(U')^2 V^2} \right) \right\} s(u, z')s(z, v) \right] \frac{\delta}{\delta s(u, v)} \end{aligned} \quad (2.14)$$

The connected parts of two- and three-dipole evolutions are subleading in N_c and it is a matter of a straightforward color algebra to see that. When acting with the NLO JIMWLK on two dipoles, the connected part arises when both dipoles are rotated by at least one J . The N_c counting of such a "dipole merging" is most easily done when $S_A(z)$ and $S_A(z')$ in the Hamiltonian are set to one. All the "dipole merging" terms generated by the operators in the NLO JIMWLK Hamiltonian are found to be subleading in N_c compared to uncorrelated terms generated by $H^{NLO \text{ dipole}}$. As a result, there are no leading $N_c \delta^2/\delta s^2$ and $(\delta^3/\delta s^3)$ terms in the dipole Hamiltonian.

Similarly to the LO case, the Hamiltonian $H^{NLO \text{ dipole}}$ is linear with respect to the dipole conjugate field $\frac{\delta}{\delta s(u, v)}$. Thanks to this important property, the dipole evolution equation

$$\frac{d\Sigma_Y^{PP}[s]}{dY} = -H^{\text{dipole}} \Sigma_Y^{PP}[s] \quad (2.15)$$

can be solved as

$$\Sigma_Y^{PP}[s] = \Sigma_{Y_0}^{PP}[s_Y] \quad (2.16)$$

where s_Y solves the NLO BK equation

$$\begin{aligned}
\frac{d}{dY} s(u, v) = & \frac{\alpha_s N_c}{2\pi^2} \int d^2 z \frac{(u-v)^2}{U^2 V^2} \left\{ 1 + \frac{\alpha_s}{4\pi} \left[b \ln(u-v)^2 \mu^2 - b \frac{U^2 - V^2}{(u-v)^2} \ln \frac{U^2}{V^2} + \left(\frac{67}{9} - \frac{\pi^3}{3} \right) N_c - \frac{10}{9} n_f - \right. \right. \\
& \left. \left. - 2N_c \ln \frac{U^2}{(u-v)^2} \ln \frac{V^2}{(u-v)^2} \right] \right\} [s(u, z) s(z, v) - s(u, v)] + \\
& + \frac{\alpha_s^2}{16N_c \pi^4} \int d^2 z d^2 z' \left[\left(-\frac{4}{(z-z')^4} + \left\{ 2 \frac{U^2(V')^2 + (U')^2 V^2 - 4(u-v)^2(z-z')^2}{(z-z')^2 [U^2(V')^2 - (U')^2 V^2]} + \right. \right. \right. \\
& \left. \left. + \frac{(u-v)^4}{U^2(V')^2 - (U')^2 V^2} \left[\frac{1}{U^2(V')^2} + \frac{1}{V^2(U')^2} \right] + \frac{(u-v)^2}{(z-z')^2} \left[\frac{1}{U^2(V')^2} - \frac{1}{(U')^2 V^2} \right] \right\} \ln \frac{U^2(V')^2}{(U')^2 V^2} \right) \times \\
& [N_c^3 s(u, z') s(z', z) s(z, v) - (z' \rightarrow z)] \\
& - \left\{ \frac{(u-v)^2}{(z-z')^2} \left[\frac{1}{U^2(V')^2} + \frac{1}{V^2(U')^2} \right] - \frac{(u-v)^4}{V^2(U')^2(V')^2 U^2} \right\} \ln \left(\frac{U^2(V')^2}{(U')^2 V^2} \right) N_c^3 s(u, z') s(z', z) s(z, v) + \\
& + N_c n_f \left\{ \frac{4}{(z-z')^4} - 2 \frac{U^2(V')^2 + (U')^2 V^2 - (u-v)^2(z-z')^2}{(z-z')^4 ((V')^2 U^2 - V^2 (U')^2)} \ln \left(\frac{U^2(V')^2}{(U')^2 V^2} \right) \right\} s(u, z') s(z, v) \Big] \quad (2.17)
\end{aligned}$$

with the initial condition

$$s_{Y_0}(x, y) = s(x, y) \quad (2.18)$$

The target average in eq.(2.1) still allows to accommodate nontrivial, non-factorized multi- s correlators $\langle s(x_1, y_1) \cdots s(x_n, y_n) \rangle_T$, which have been recently argued [18, 19] to be of relevance to various two-particle correlations, such as the "ridge".

Further simplification is achieved if one assumes that the dipoles scatter on the target independently. This amounts to factorization of the target averages of the dipole s -matrices

$$\langle s(x, y) s(u, v) \rangle_T = \langle s(x, y) \rangle_T \langle s(u, v) \rangle_T \quad (2.19)$$

With this assumption, one replaces the ensemble average over target fields with a fixed initial function $s_{Y_0}(x, y)$. We refer to this factorization property as the target mean field approximation. Within the target mean field approximation

$$\langle \Sigma_0^{PP}[s_Y] \rangle_T = \Sigma_0^{PP}[\langle s_Y \rangle_T] . \quad (2.20)$$

As has been stressed in the past, this mean field approximation does not follow from the dipole model approximation for the evolution kernel eq.(2.15), but is an additional assumption about the properties of the target.

III. DIFFRACTIVE DISSOCIATION

We will be interested in identifying the evolution governing the diffractive observables both with respect to the total rapidity of the process Y and the rapidity gap. We now consider processes where the projectile diffracts within rapidity interval Y_P . This interval is not necessarily small, so this type of observable can be evolved independently over the total rapidity Y and the width of the diffractive interval Y_P . The target can either scatter elastically or can in principle also diffract. We consider here the process where the scattering on the target side is elastic with the minimal gap $Y_{gap} = Y - Y_P$.

In ref. [16] we have developed a formalism that makes it possible to generalize (2.1) for semi-inclusive processes. Particularly, for elastic/diffractive processes one has to introduce two independent S -matrix variables, S and \bar{S} for the amplitude and its conjugate. For the observable in question the cross section reads

$$N_E^{D, Y_P} = \int DS D\bar{S} (1 - \Sigma_{Y_P}^{PP}[S] - \Sigma_{Y_P}^{PP}[\bar{S}^\dagger] + \Sigma_{Y_P}^{PP}[\bar{S}^\dagger S]) W_{Y_{gap}}^T[S] W_{Y_{gap}}^T[\bar{S}]. \quad (3.1)$$

What we have here is the target evolved through the gap Y_{gap} both in the amplitude and its conjugate, while the projectile is evolved inclusively through the diffractive interval Y_P . The derivation of this result did not rely on any explicit form of the evolution Hamiltonian and is thus valid for the case of the NLO JIMWLK.

We would like now to project this result into the dipole picture at large N_c . At leading order, this observable in the dipole limit has been discussed by Kovchegov and Levin [13]. In the dipole limit the evolution of Σ^{PP} with respect

to the diffractive interval at fixed Y_{gap} is given by eq. (2.16). Within the gap, due to independent target averaging over S and \bar{S} in eq. (3.1), the "composite" dipole made of $\bar{S}^\dagger S$ factorizes into the product of two

$$\frac{1}{N_c} \langle \text{tr}[\bar{S}^\dagger(x)S(x)S^\dagger(y)\bar{S}(y)] \rangle_T = \langle \frac{1}{N_c} \text{tr}[S(x)S^\dagger(y)] \rangle_T \langle \frac{1}{N_c} \text{tr}[\bar{S}(y)\bar{S}^\dagger(x)] \rangle_T \quad (3.2)$$

with each dipole evolving according to its own NLO BK equation (2.17). Similarly to the LO case, we have

$$N_E^{D,Y_P} = 1 - 2\mathcal{S}(Y) + \Sigma_0^{PP}[s_{Y_P,Y_{gap}}^{el}] \quad (3.3)$$

where $s_{Y_P,Y_{gap}}^{el}$ is obtained by first solving the NLO BK equation with respect to Y_{gap} with the initial condition

$$s_{Y_{gap}=0}(x,y) = s(x,y) \quad (3.4)$$

and then evolving by Y_P with the very same equation for s^{el} with the initial condition

$$s_{Y_P=0,Y_{gap}}^{el} = s_{Y_{gap}}^2. \quad (3.5)$$

As has been already noticed in [16], the derivation in [13] was originally done for a single dipole projectile, $\Sigma^{PP}[s] = s$. Thanks to the fact that at both LO and NLO, all the dipoles evolve independently, Eq. (3.3) provides generalization to a more complex projectile wave function.

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